



Engineering Drawing & Graphics

PROJECTION OF LINES

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PROJECTION OF POINTS & LINES

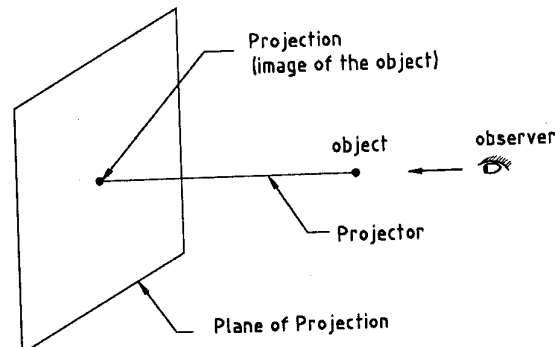
ABBREVIATIONS

Following abbreviations, symbols and notation will be used in the entire course.

VP	- Vertical plane
HP	- Horizontal plane
XY	- Reference line
X_1Y_2, X_2Y_2	- Auxiliary reference plane
HT	- Horizontal trace
VT	- Vertical trace
A,B,C etc.	- to represent a point/line ends/solid corners in space
a,b,c etc.	- to represent top view of a point/line ends/solid corners
a', b', c' etc.	- to represent front view of a point/line ends/solid corners
AVP	- Auxiliary vertical plane
AIP	- Auxiliary inclined plane
θ	- True inclination of a line with HP
ϕ	- True inclination of a line with VP
α	- Apparent inclination of a line with HP
β	- Apparent inclination of a line with VP

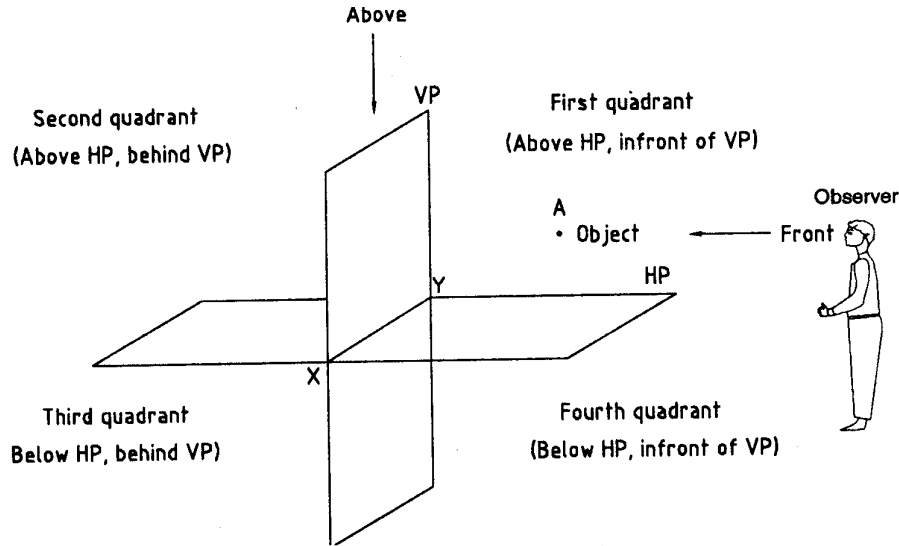
INTRODUCTION TO PROJECTION

Projecting the image of an object to the plane of projection is known as **projection**. The object may be a point, line, plane, solid, machine component or a building. Consider the following illustration to project the image of an object on to a plane.

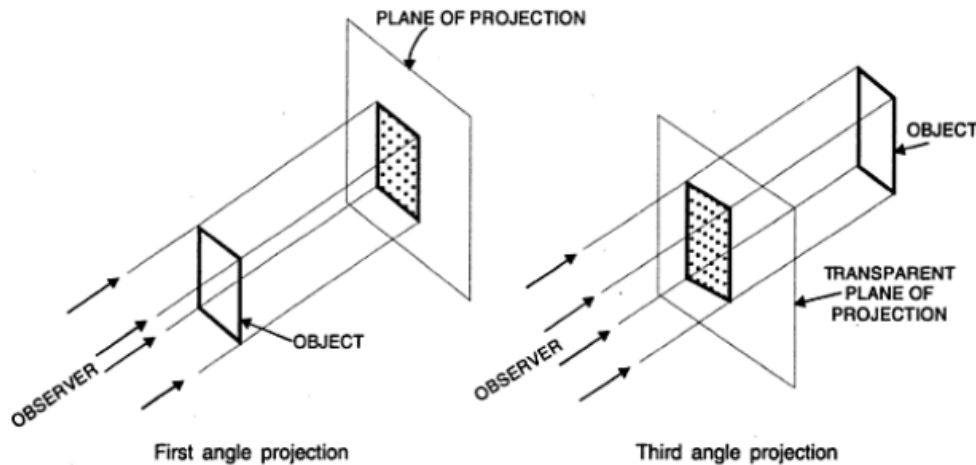


In engineering drawing practice, two principal planes are used to get the projection of object as shown in figure. They are

- (1) Vertical plane (VP) which is assumed to be placed vertically. The front view of the object is projected onto this plane.
- (2) Horizontal plane (HP) which is assumed to be placed horizontally. The top view of the object is projected onto this plane.



When an object is assumed to be placed in first quadrant, the projection method followed is called as **first angle projection**. In this method, the object is placed between the observer and the plane of projection.



When the object is assumed to be placed in third quadrant, the projection method followed is called as **third angle projection**. In this method, the plane of projection lies between the object and the observer.

Important Observations

- (1) First angle projection is used in USA, whereas Third angle projection is used in European countries.
- (2) Bureau of Indian Standards has recommended that only **first angle projection** method should be used in our country. However, third angle projection system is still accepted

in industrial organizations. Therefore, engineering students must have knowledge of both methods of projections.

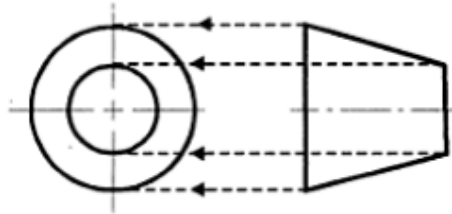
(3) An engineer should be comfortable with both, First angle and Third angle projection method

(4) *In this entire course, First angle projection method is used.*

FIRST ANGLE VS. THIRD ANGLE PROJECTION METHOD

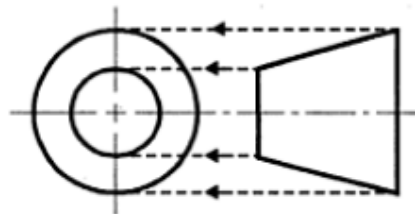
First Angle Method

1. The object lies In between the observer and the plane of projection. The plane of projection Is always behind the object.
2. The object is assumed to be placed in first quadrant.
3. The front view or the elevation Is always above the top view or the plan.
4. The right hand end view/side view Is drawn to the left and left hand end view Is drawn to the right.
5. The plane of projection may or may not be transparent.
6. It Is represented by the following symbol:



Third Angle Method

1. Between the observer and the object ore transparent planes of projection. The plane of projection Is always In front of the object.
2. The projections are drawn assuming that Uie object Is situated In third quadrant.
3. The front view Is always below the top view.
4. The right hand end view is drawn to the right and left hand end view Is drawn to the left.
5. The plane of projection Is always transparent.
6. It Is represented by the following symbol :

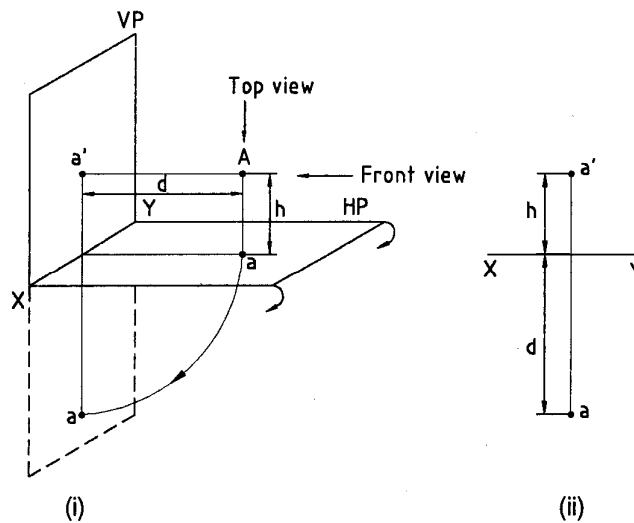


- (1) In drawing practice, capital letters A,B,C etc are used to represent objects in space.
- (2) Their *top views* are represented by small letters a,b,c etc.
- (3) The *front views* are represented by small letters with dashes a',b',c', etc.
- (4) These letters are used to represent a point, ends of a straight line, corners of solid etc.
- (5) Actual projections in top and front views are drawn in thick lines, construction lines and projectors are drawn using thin lines.
- (6) Top view is also known as *plan* and front view is known as *elevation*.

PROJECTION OF A POINT IN FIRST QUADRANT

Consider a point A placed in the first quadrant. This is at a height h mm above HP, at a distance d mm in front of VP. Its front view a' is projected onto VP and the top view a is projected onto HP.

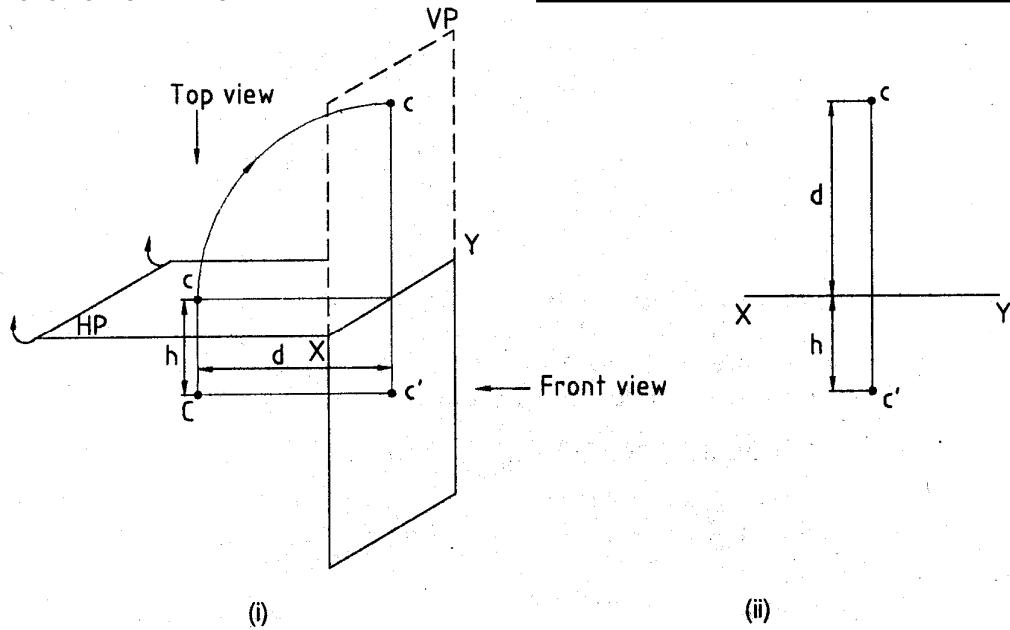
Now the HP is rotated in the clockwise direction for 90° and is obtained in vertical position. The projections will be seen as given in Fig.(ii).



It is drawn with reference to XY line. Mark a point a' at a height h mm above XY, and a at a distance d mm below XY. The projector joining a' and a is always perpendicular to XY.

PROJECTION OF POINT IN THIRD QUADRANT

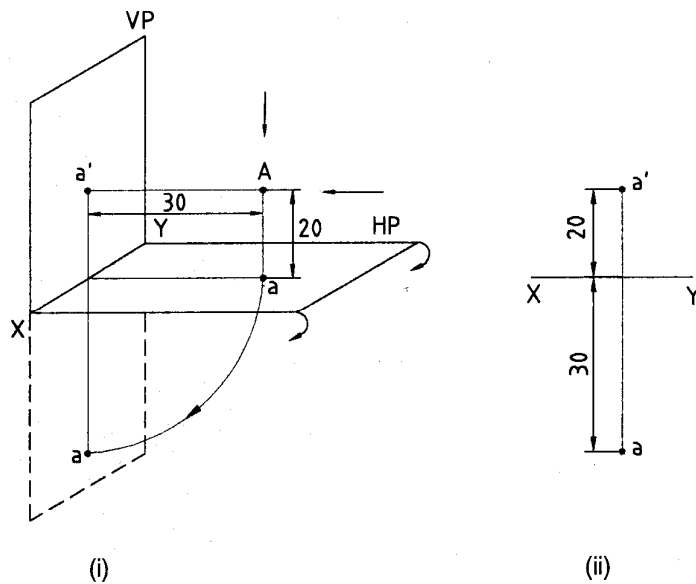
Consider a point C placed in the third quadrant. The point is at a height h mm below HP and a distance d mm behind VP. Its front view c' is projected onto VP and the top view c is projected onto HP.



Now the HP is rotated in the clockwise direction for 90° and is obtained in vertical position. The projections will be seen as given in Fig.(ii). It is drawn with reference to XY line. Mark a point c' at a height h mm below XY and c at a distance d mm above XY.

EXAMPLE

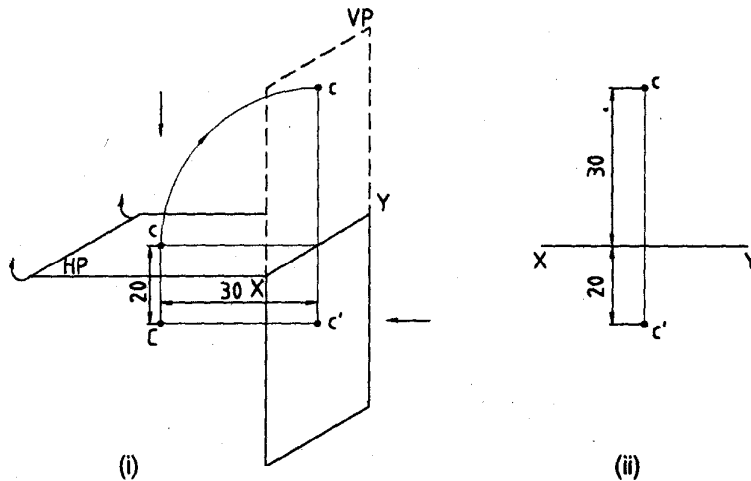
A point A is 20 mm above HP and 30 mm in front of VP. Draw its projection.



To draw the projections

Draw the reference line XY. Mark a point a' at a distance of 20 mm above XY. Through this point draw a perpendicular line to XY and mark the top view a at a distance of 300 mm below XY.

A point C is 20mm below HP and 30 mm behind VP. Draw its projection.



To draw the projections

Draw the reference line XY. Mark a point c' at a distance of 20 mm below XY. Through this point draw a perpendicular line to XY and mark the top view c at a distance of 30 mm above XY.

PROJECTION OF STRAIGHT LINES

A straight line is the shortest route to join any two given points. It is a one-dimensional object having only length (l).

The projection of straight line are obtained by joining the top and front views of the respective end points of the line. The actual length of the straight line is known as true length (TL).

Example

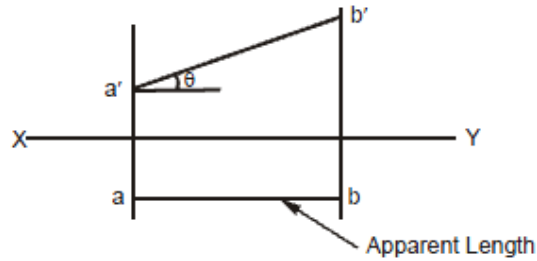
Explain true length concept with an example.

Solution

Hold a pencil in your hand and open a note-book in such a way that some pages are on the table and some pages are held perpendicular to the table top. Consider these as two planes HP and VP.

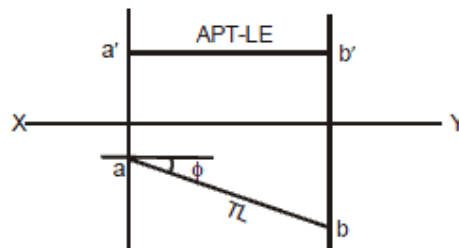
Put one end of the pencil on the HP and let it make some angle with HP. Make the pencil parallel to VP (imagine the pencil to be a line).

Under the condition if the line is projected on HP and VP, its projections will be as shown in following figure.



The elevation of the line $a'b'$ will be true length, the angle θ will also be true angle but plan ab will be shorter than true length. The length ab is a function of angle θ . If θ is increased, ab will be reduced further.

Now consider the pencil parallel to HP and making some angle ϕ with VP. Its projections will be as shown in following figure.



The length of plan is True length TL , the angle made by this plan with VP , i.e. with XY line is true angle ϕ , and the length of elevation is a function of angle ϕ . Elevation is parallel to XY (in second case, the line is given parallel to HP , while in this case the elevation is a line parallel to XY and the plan of the line is TL , the angle made ϕ is true angle).

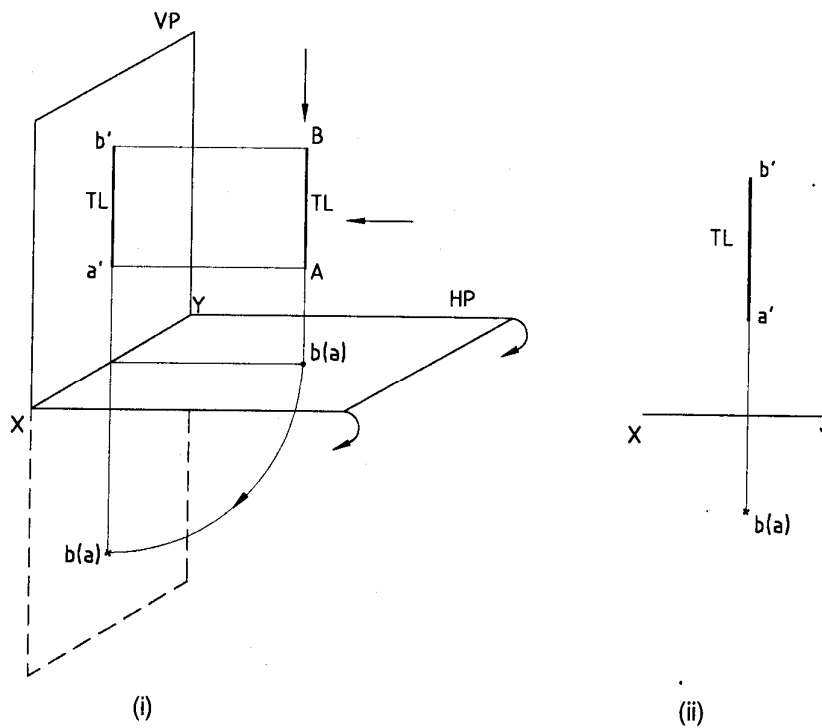
From the example, we see that in first case when the line was parallel to VP , the elevation of the line is TL , the angle made, i.e. θ , is also true angle and the plan is a line parallel to $X-Y$.

Hence, learn a rule that if the projections of the line are given and if one of the views (plan or elevation) is parallel to XY , the other view **MUST BE** true length and angle made by this TL will be true angle. Its reverse is also true, i.e. from among the views of a line, if one view is TL , the other view **MUST BE** parallel to $X-Y$.

A straight line is placed with reference to the planes of projections in the following positions.

1. Line perpendicular to HP and parallel to VP .
2. Line perpendicular to VP and parallel to HP .
3. Line parallel to both HP and VP .
4. Line inclined to HP and parallel to VP .
5. Line inclined to VP and parallel to HP .
6. Line inclined to both HP and VP .

In first angle projection method, the line is assumed to be placed in the first quadrant. The projections of the line in the above mentioned positions are discussed below.

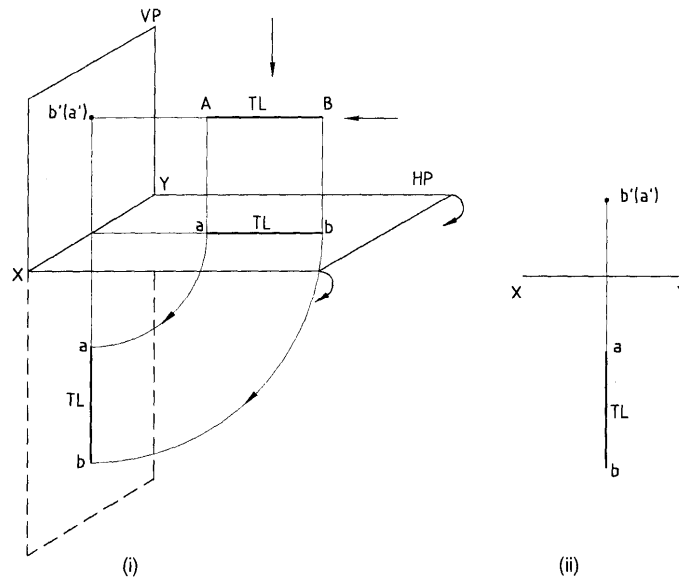


Consider a straight line AB kept perpendicular to HP and parallel to VP Fig.(i). Its front view is projected onto VP which is a line having true length. The top view is projected onto HP which is a point one end b of which is visible while the other end a is invisible and is enclosed within ().

Now the HP is rotated in the clockwise direction through 90^0 and is obtained in the vertical position. The projections obtained are seen as given in Fig. It is drawn with reference to the XY line as follows.

1. Draw the XY line.
2. Draw the front view a'b'. which is a line perpendicular to XY and having true length (TL).
3. Projected the top view ab. The end b is visible and the invisible end a is marked inside ().

Projection of a Line kept Perpendicular to VP and Parallel to HP

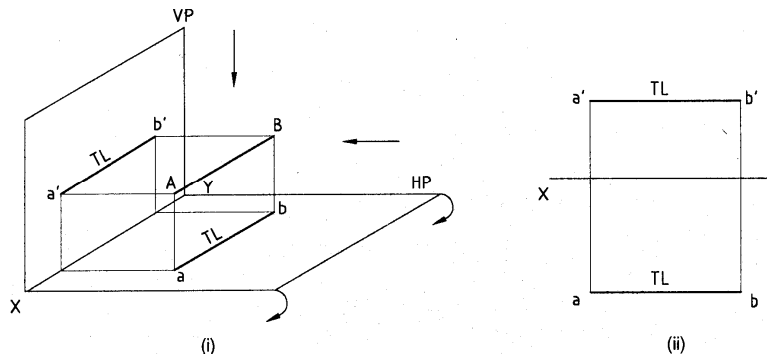


Consider a straight line AB kept perpendicular to VP and parallel to HP Fig.(i). Its top view is projected onto HP which is a line having true length (TL). The front view is projected onto VP which is point, the end b' of which is visible and the other end a' is invisible which is shown enclosed in ().

Now the HP is rotated in the clockwise direction through 90^0 and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with reference to the XY line as follows.

1. Draw the XY line.
2. Draw the top view ab , a line perpendicular to XY and having true length (TL).
3. Projected the front view $a'b'$. The end b' is visible and the invisible end a' is marked inside ().

Projection of a Line kept Parallel to Both HP and VP



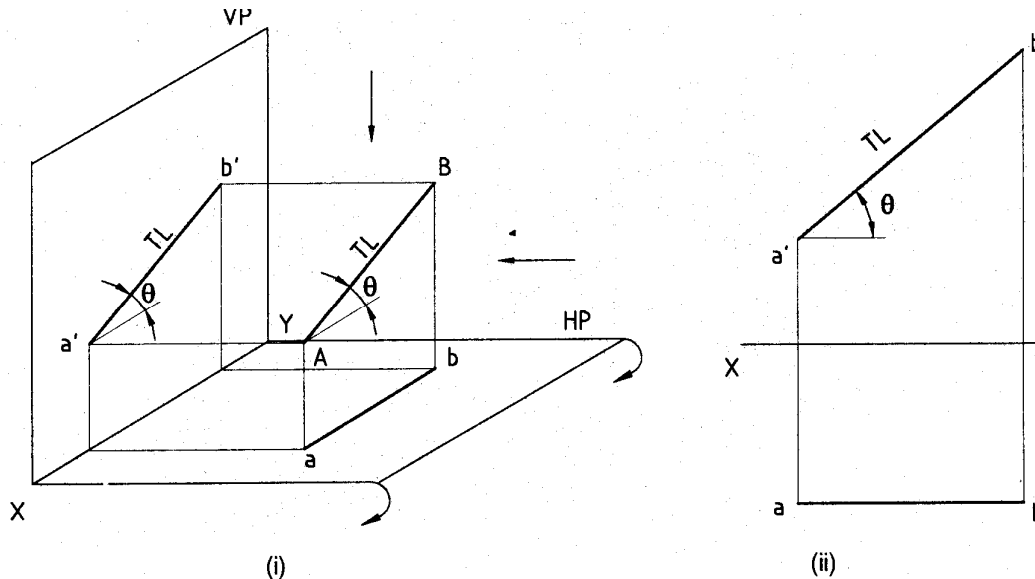
Consider a straight line AB kept parallel to both HP and VP Fig.(i). Its front view is projected onto VP which is a line having true length (TL). The top view is projected onto HP which is also a line having true length.

Now the HP is rotated in the clockwise direction through an angle of 90^0 and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with references to the XY line as follows:

1. Draw the XY line.
2. Draw the front view $a'b'$, a line parallel to XY and having true length (TL)
3. Project the top view ab which is also a line parallel to XY having true length (TL).

Projection of a Line kept inclined to HP and Parallel to VP

Consider a straight line AB kept inclined to HP and parallel to VP Fig.(i). Its front view is projected onto VP which is an inclined line at an angle θ to XY and having true length (TL). The top view is projected onto HP which is also a line but smaller than the true length and parallel to XY. The inclination of the line with HP is always represented by the symbol θ .

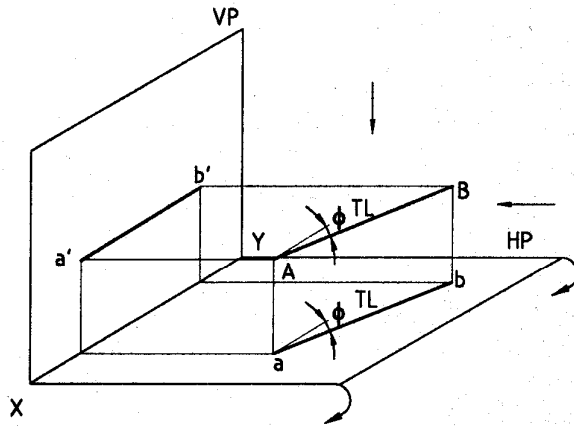


Now the HP is rotated in the clockwise direction through 90^0 and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with reference to the XY line as follows:

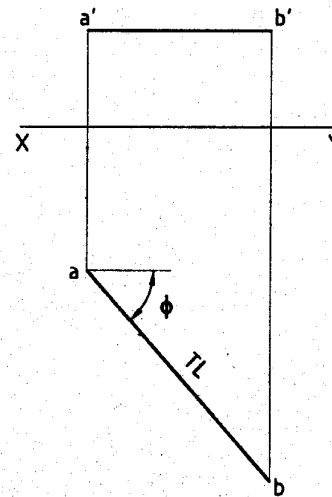
1. Draw the XY line.
2. Draw the front view $a'b'$, a line inclined at an angle θ to XY having true length (TL).
3. Project the top view ab which is also a line parallel to XY and smaller than true length.

Projection of a Line kept Inclined to VP and Parallel to HP

Consider a straight line AB kept inclined to VP and parallel to HP Fig.(i). Its top view is projected onto HP which is a line inclined at an angle ϕ to XY and having true length (TL). The front view is projected onto VP which is also a line but smaller than true length and is parallel to XY. The inclination of the line with VP is always represented by the symbol ϕ .



(i)



(ii)

Now the HP is rotated in the clockwise direction through an angle of 90^0 and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with reference to the XY line as follows:

1. Draw the XY line
2. Draw the top view ab , a line inclined at an angle ϕ to XY and having true length (TL).
3. Project the front view $a'b'$, which is also a line parallel to XY but smaller than true length.

TRACE OF A LINE

The point of intersection or meeting of a line with the reference plane, extended if necessary, is known as the **trace of a line**. The point of intersection of a line with the HP is known as the **horizontal trace**, represented by HT and that with the VP is known as the **vertical trace**, represented by VT. No trace is obtained when a line is kept parallel to a reference plane.

If the line is given parallel to a plane, it will never intersect that plane and, therefore, no trace of the line on that plane.

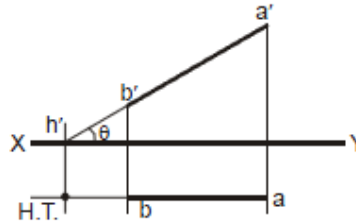
If the line is given parallel to VP and inclined to HP, only HT will be obtained and no VT.

If the line is given parallel to HP and inclined to VP, only VT will be obtained and no HT.

If the line is given parallel to both the planes, neither HT nor VT will be obtained.

Rule For obtaining HT of an inclined line AB

For obtaining HT of an inclined line AB , inclined to HP, extend the elevation $a'b'$ towards $X-Y$ line till it intersects $X-Y$ (In some problems, depending upon the position of the given line, the elevation might intersect $X-Y$ line in natural course and it need not be extended. This may happen when one end of the line is above HP and the other end below HP. In this also the point of intersection of elevation with $X-Y$ will be h' .) The point of intersection of the extended elevation with $X-Y$ is h' .



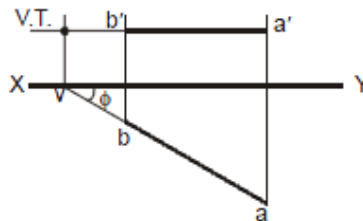
Now from h' , draw a line perpendicular to $X-Y$. Then extend the plan ab in its own direction, till it intersects the perpendicular from h' .

Rule For obtaining VT of an inclined line AB

The point of intersection of extended plan ab (in some cases the plan need not be extended but the perpendicular from h' will intersect the plan ab) and the perpendicular from h' will be HT.

For obtaining the VT the procedure is reversed

- (a) Extend the plan towards $X-Y$ line till it intersects $X-Y$. The point of intersection is v .
- (b) Draw a line perpendicular to $X-Y$ from h .
- (c) Extend the elevation in its own direction, so that it intersects the perpendicular drawn from h .



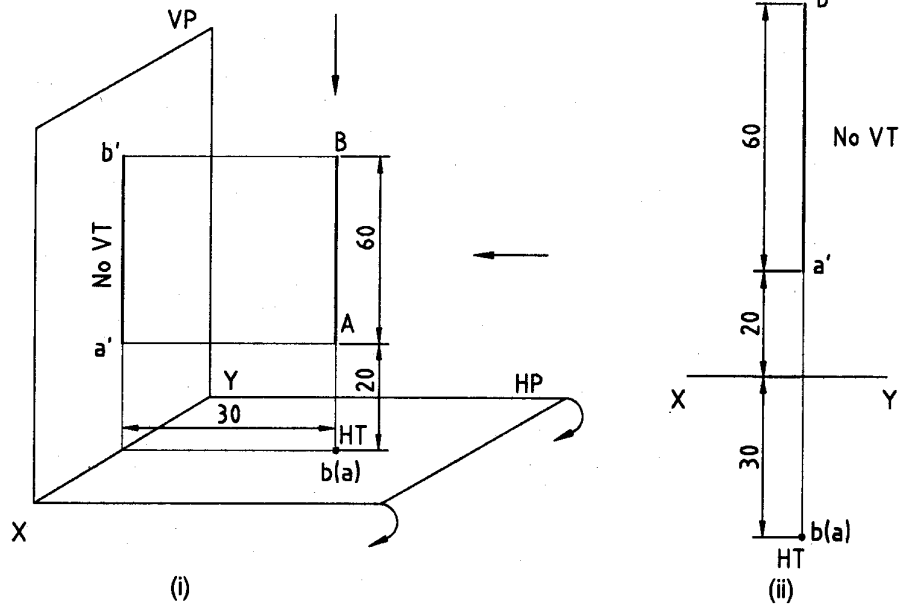
- (d) The point of intersection is VT. (In some cases, the elevation need not be extended and the perpendicular from h will intersect the elevation. This point of intersection is VT)

EXAMPLE

A line AB 60 mm long has its end 20 mm above HP and 30 mm in front of VP. The line is kept perpendicular to HP and parallel to VP. Draw its projections. Also mark the traces.

SOLUTION

Assume that end A of the line is nearer to HP. The front view $a'b'$ is a line having true length. The top view is a point, the end b of which is visible and a is invisible which is enclosed in $()$. The line is extended to meet HP to obtained the horizontal trace (HT). No vertical trace (VT) is obtained because the line is kept parallel to VP.



The projections obtained are drawn with reference to XY line as shown in Fig.12(ii).

(You should not draw diagram (i) in the examination. This diagram is given for explanation only.)

1. Mark the projections of the end A by considering it as a point. Its front view a' is 20 mm above XY and the top view a is 30 mm below XY.
2. The front view of the line $a'b'$ is obtained by drawing a line perpendicular to XY from a' and having a length of 60 mm.
3. Top view of the line is obtained by projecting the other end b which coincides with a . The invisible end a is enclosed in ().
4. The horizontal trace (HT) is marked coinciding with the top view of the line. NO vertical trace (VT) is obtained.

EXAMPLE

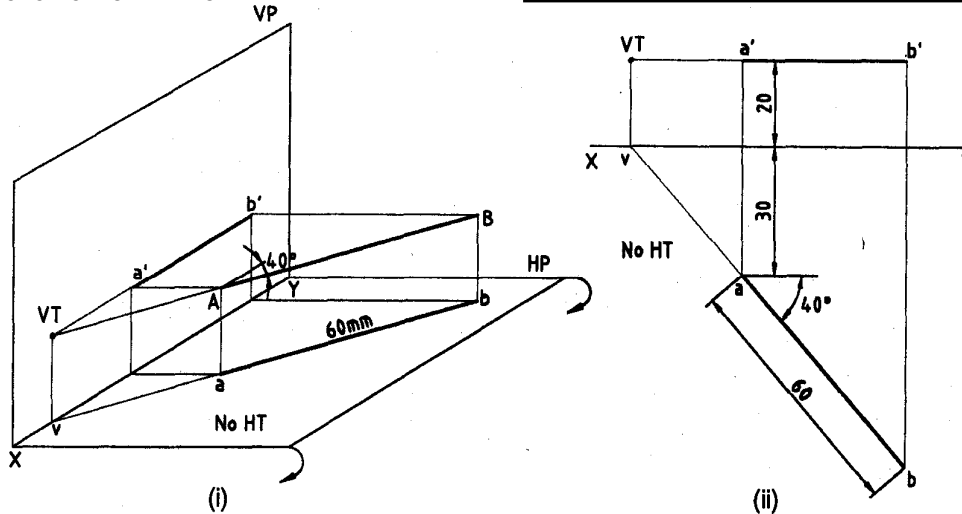
A line AB 60 mm long has its end A 20 mm above HP and 30 mm in front of VP. The line is inclined at 40° to VP and parallel to HP. Draw its projections. Also mark the traces.

SOLUTION

The top view ab is a line inclined at an angle of 40° to XY and having true length. Its front view $a'b'$ is parallel to XY and smaller than true length.

The line is extended to meet VP to obtained the vertical trace (VT). No horizontal trace (HT) is obtained because the line is kept parallel to HP.

The projections obtained are drawn with reference to XY line as shown in Fig.(ii).



1. Mark the projections of end A by considering it as a point. Its front view a' is 20 mm above XY and top view a is 30 mm below XY.
2. The top view of the line ab is obtained by drawing a line inclined at an angle 40° to XY from a and having a length of 60 mm.
3. The front view of the line $a'b'$ is obtained by drawing a line parallel to XY from a' and drawing a vertical line (projector) from b . It is parallel to XY and smaller than the true length.
4. To mark the vertical trace (VT) the top view of the line ab is extended to intersect with XY line at v . Then by drawing a vertical line from v and a horizontal line from $a'b'$, the VT is located. No horizontal trace (HT) is obtained.

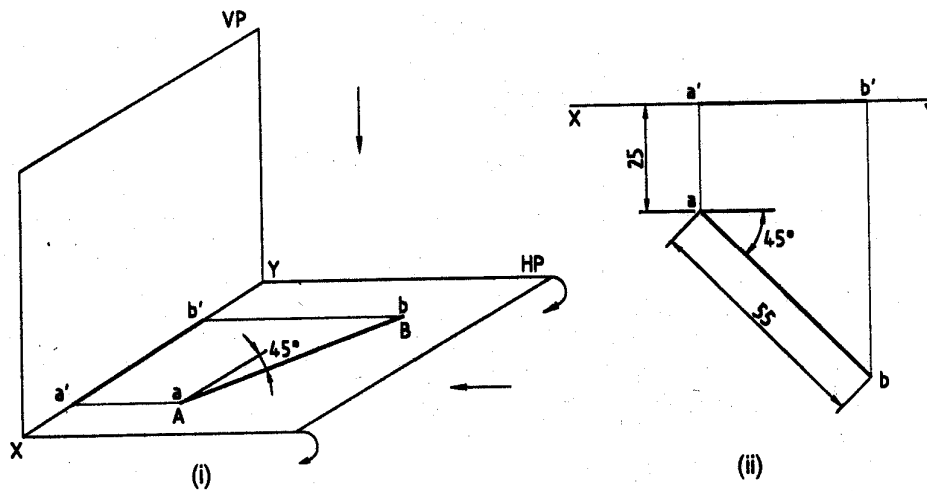
EXAMPLE

A line AB 55 mm long has its end 25 mm in front of VP and in HP. The line is inclined at 45° to VP. Draw its projections.

SOLUTION

The projections obtained are drawn with reference to the XY line as shown in Fig.(ii).

1. Mark the projections of the end A by considering it as a point. Its top view A is 25 mm below XY and front view a' is on the XY line.
2. The top view of the line ab is obtained by drawing a line inclined at 45° to XY from a and having a length of 55 mm.
3. The front view of the line $a'b'$ is obtained by drawing a line on XY from a' and drawing a vertical line from b . It is smaller than the true length.

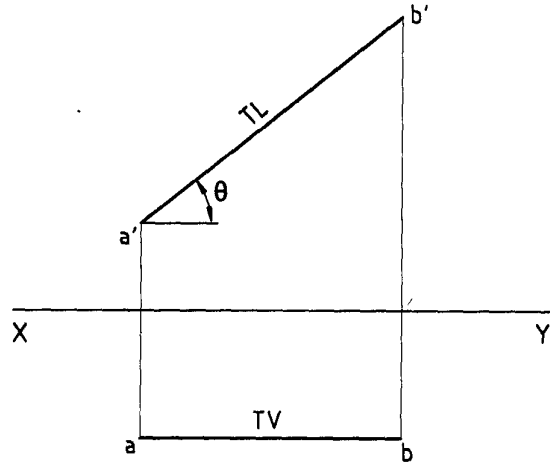


When a line is inclined to one plane and parallel to the other plane.

(a) When a line is inclined to HP and parallel to VP

The projections are usually obtained as follows. There are three variables namely TL, θ and TV marked in the drawing. In a problem usually any two variable values will be given and the third variable value can be obtained graphically by completing the drawing as mentioned below. Draw the projections a' and a of the given end A.

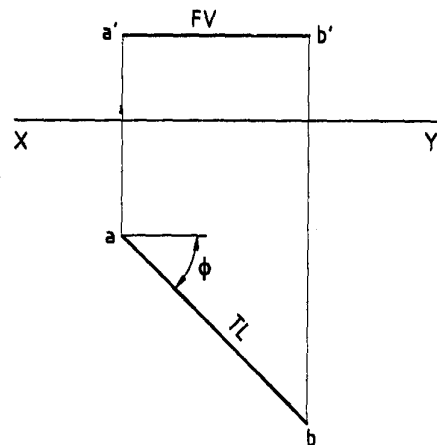
- (i) When TL and θ are given. Draw the front view $a'b'$ from a' using TL and θ . Top view (TV) ab is projected and obtained by drawing a line parallel to the XY line and a vertical line (projector) from b' .
- (ii) When TL and TV are given. Draw the top view ab using TV parallel to XY line. Draw the vertical line (projector) from b . Using TL as radius and a' as centre, mark a point in the vertical line to get b' . Join a' and b' to complete the front view $a'b'$ of the line. The inclination of $a'b'$ with XY is measured to get θ .
- (iii) When TV and θ are given. Draw the top view using the length of TV parallel to the XY line. Draw the vertical line (projector) from b . Using the angle θ , draw a line which intersects the projector at b' . Join a' and b' to front view $a'b'$ of the line. The length of $a'b'$ is measured to get TL.



TL - True length
 TV - Top view
 θ - Inclination with HP

(b) When a line is inclined to VP and parallel to HP

The projections are usually obtained as follows.



TL - True length
 FV - Front view
 ϕ - Inclination with VP

There are three variables namely TL, ϕ and FV marked in the drawing. In a problem usually any two variable values will be given the third variable value can be obtained graphically by completing the drawing as mentioned below. Draw the projections a' and a of the given end A.

- (i) When TL and ϕ are given. Draw the top view ab using TL and ϕ . The front view (FV) $a'b'$ is projected and obtained by drawing a line parallel to XY and a vertical line (projector) from b .
- (ii) When TL and FV are given. draw the front view $a'b'$ using FV parallel to the XY line. Draw the vertical line (projector) from b' . Using TL as radius and a as centre, mark a point in the vertical line to get b . Join a and b to top view ab of the line. The inclination of ab with XY is measured to get ϕ .
- (iii) When FV and ϕ are given. Draw the front view $a'b'$ using FV parallel to the XY line. Draw the vertical line (projector) from b' . Using the angle ϕ , draw a line

which intersects with the projector at b. Join a and b to complete the top view ab of the line. The length of ab is measured to get TL.

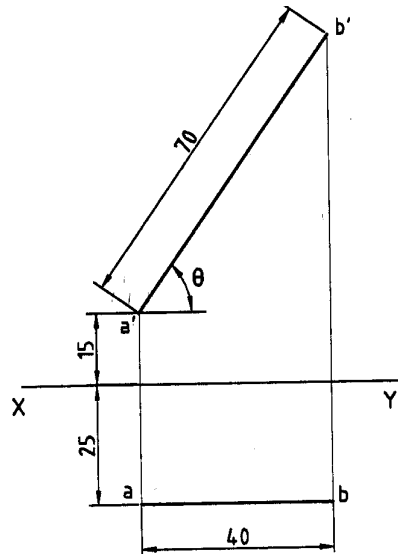
EXAMPLE

A line AB 70 mm long has its end A 15 mm above HP and 25 mm in front of VP. Its top view (plan) has a length of 40 mm. Draw its projections and find the inclination of the line with HP.

SOLUTION

The projections of the line are drawn with reference to the XY line as follows:

1. Mark the projections of end A by considering it as a point. Its front view a' is 15 mm above XY and top view a is 25 mm below XY.
2. The top view of the line ab is drawn parallel to XY to the given length of 40 mm.
3. Draw a vertical line (projector) from b.
4. Using true length 70 mm as radius and a' as centre, mark a point in the vertical line to get b'.
5. The inclination of a'b' with XY is measured to get θ .

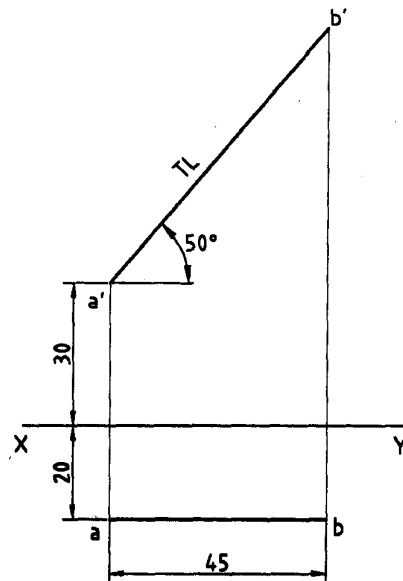


ANSWER

$\theta = 55^\circ$

EXAMPLE

A line AB has its end 30 mm above HP and 20 mm front of VP. Its plan has a length of 45 mm. The line is inclined at 50° to HP and parallel to VP. Draw its projections and find the true length of the line.



ANSWER
TRUE LENGTH = 70mm

The projections of the line are drawn with reference to the XY line as follows.

1. Mark the projections of end A by considering it as a point. Its front view a' is 30 mm above XY and top view a is 20 mm below XY.
2. The top view a is 20 mm below XY.
3. Draw a vertical line (projector) from b .
4. Using the angle 50° , draw a line from a' to get the front view $a'b'$ of the line.
5. The length of $a'b'$ is measured to get the true length of the line.

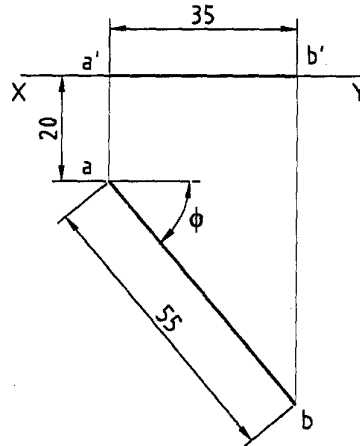
EXAMPLE

A line AB 55 mm long is in HP and inclined to VP. The end A is 20 mm in front of VP. The length of front view is 35 mm. Draw the projections of the line and also find the inclination of the line with VP.

SOLUTION

The projections of the line are drawn with reference to the XY line as follows:

1. Mark the projections of end A by considering it as a point. Its front view a' is on XY and top view a is 20 mm below XY.
2. The front view $a'b'$ of the line is drawn on XY for a length of 35 mm.
3. Draw a vertical line (projector) from b' .
4. Using true length 55 mm as radius and a as centre, mark a point in the vertical line to get b . Join a and b to complete the top view ab of the line.
5. The inclination of ab with XY is measured to get ϕ .



ANSWER

$$\phi = 50^\circ$$

Projection of a Line kept Inclined to Both HP and VP

When a line is placed inclined to both HP and VP, its projections obtained in top and front views are smaller than the true length of the line and inclined to the XY line. So it is impossible to project and draw the top or front view of the line directly. Any one of the following methods may be used to draw the projections.

1. Rotating line method
2. Rotating trapezoidal plane method
3. Auxiliary plane method

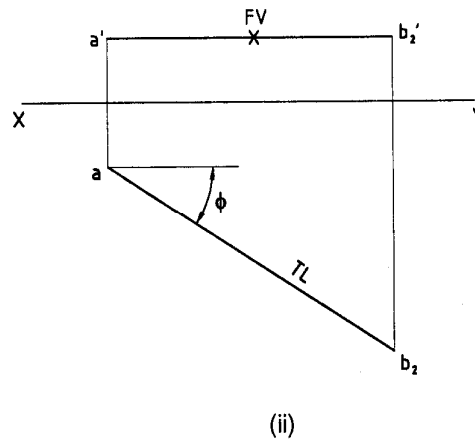
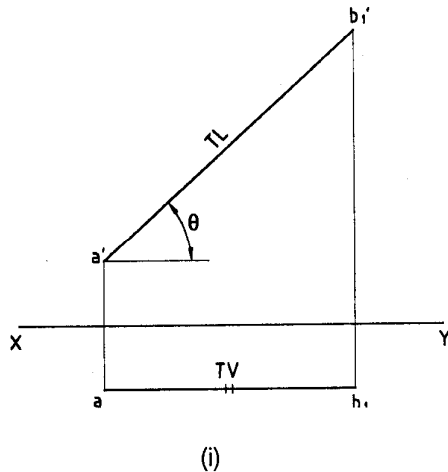
1. Rotating Line Method

Consider a line AB is placed inclined at θ to HP and ϕ to VP. Draw its projections assuming that the line is placed in the first quadrant. The following steps are to find the top view (plan) and front view (elevation) lengths and then, they are rotated to the required position to represent the projections of the line in the given position.

Mark the projections of the end A by considering it as a point. Its front view a' will be obtained above XY and top view a will be obtained below the XY line.

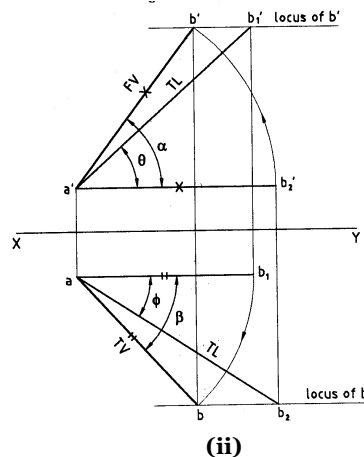
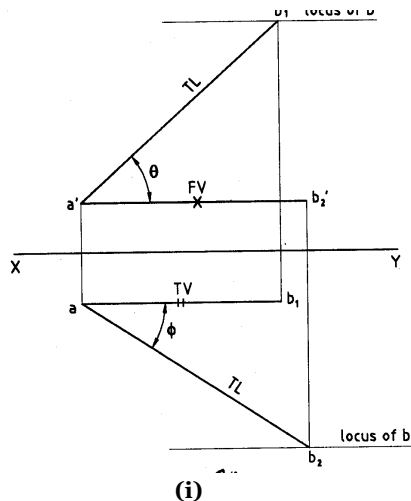
Step 1: Assume that the line is kept inclined to HP and a re parallel to VP. Draw the front view $a'b_1'$, it is a line inclined at θ to XY ad having true length (TL). Project and get the top view ab_1 length which is parallel to XY line. Then this will be rotated to the required position. [Figure.(i)].

Step2: Assume the line is kept inclined to VP and parallel to HP. Draw the top view ab_2 , it is a line inclined at ϕ to XY and having true length (TL). Project and get the front view ab_2' length which is parallel to XY line. Then this will be rotated to the required position. [Figure (ii)].



Step3: Draw the locus of the other end B of the line in top and front views. Draw the locus of the front view b' as a line passing through b_1' and parallel to XY line. Draw the locus of the top view b as a line passing through b_2 and parallel to XY line [Fig.(iii)]. Note that step 1 and step 2 are shown together.

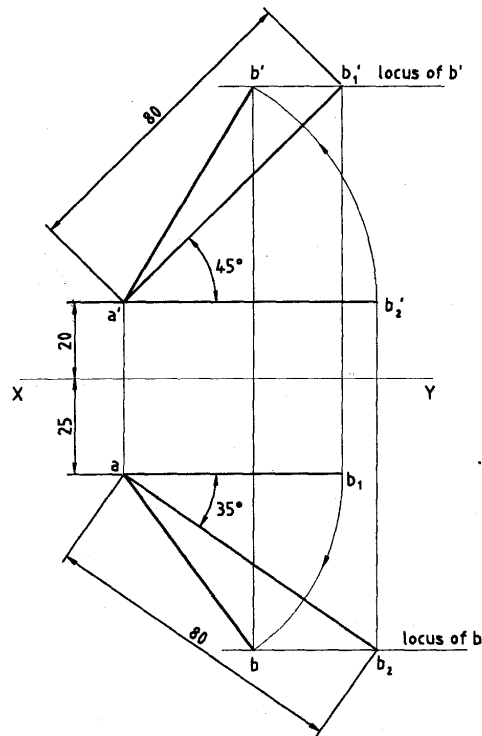
Step4: Rotate the top view ab_1 and front view $a'b_2'$ to the required position. Take a as centre, top view length ab_1 as radius, draw an arc to intersect with the locus of b at b . Join a and b to get the top view ab of the line in required position. Taking a' as centre, front view $a'b_2'$ as radius, draw an arc to intersect the locus of b' at b' . Join a' and b' to get the front view $a'b'$ of the line in required position [Fig.(iv)].



Check the drawing obtained, by drawing the projector for the end B by joining b' and b which is a line always perpendicular to XY line.

EXAMPLE

A line AB 80 mm long has its end A 20 mm above HP and 25 mm in front of VP. The line is inclined at 45° to HP and 35° to VP. Draw its projections.

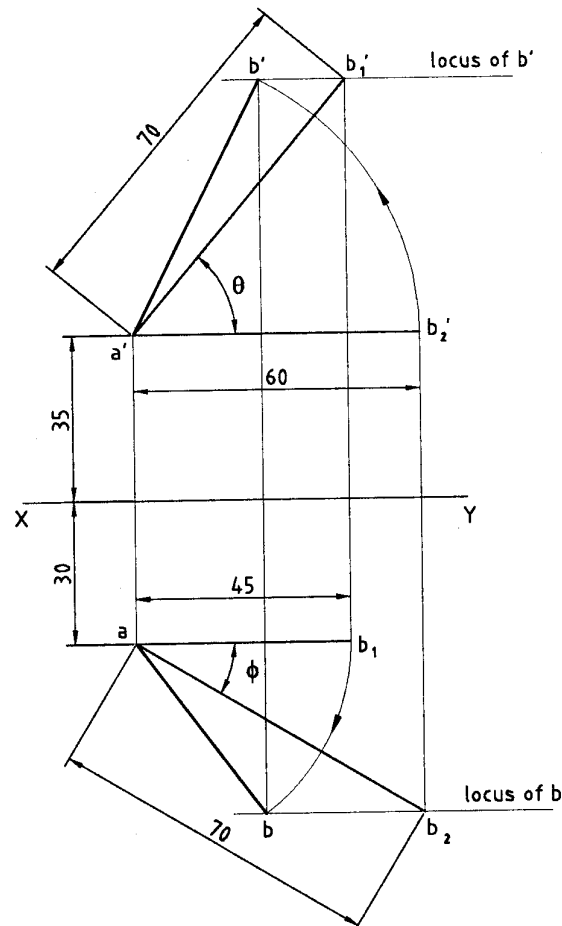


Mark the projections of end A by considering it as a point. Its front view a' is 20 mm above XY and top view a is 25 mm below the XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view $a'b_1'$, a line inclined at 45° to Xy line and having a length of 80 mm. Project and get the top view ab_1 length which is parallel to XY line.
2. Assume that the line kept inclined to VP and parallel to HP. Draw the top view ab_2 , a line inclined at 35° to XY line and having a length of 80 mm. Project and get the front view $a'b_2'$ length which is also parallel to XY line.
3. Draw the locus of the other end B of the line in top and front views. Draw the locus of b' which is a line passing through b_1' and parallel to XY line. Also draw the locus of b which is a line passing through b_2 and parallel to XY line.
4. Rotate the top view ab_1 and front view $a'b_2'$ to the required position. Take a as centre, top view length ab_1 as radius, draw an arc to intersect the locus of b at b . Join a and b to get the top view ab of the line. Take a' as centre, front view length $a'b_2'$ as radius, draw an arc to intersect the locus of b' at b' . Join a' and b' to get the front view $a'b'$ of the line.
5. Check the result obtained by drawing the projector joining b' and b which should be a vertical line.

A line AB 70 mm long, has its end A 35 mm above HP and 30 mm in front of VP . The top view and front view has a length of 45 mm and 60 mm respectively. Draw its projections.

SOLUTION



ANSWERS

$\theta = 50^\circ$

$\phi = 31^\circ$

Mark the projections of the end A by considering it as a point. Its front view a' is 35 mm above XY and top view a is 30 mm below XY line.

1. Assume that the line is kept inclined to HP and parallel to VP . In this case, considering the given data the top view ab_1 can be drawn parallel to XY and having a length of 45 mm. Draw a vertical (projector) through b_1 . Using true length 70 mm as radius and a' as centre, draw an arc to intersect the vertical line through b_1 to get b'_1 which represents the true length of the line. The inclination of $a'b'_1$ to XY is the inclination of the line with $HP(\theta)$.
2. Assume that the line is kept inclined to VP and parallel to HP . In this case, considering the given data, the front view $a'b'_2$ can be drawn parallel to XY and having a length of 60 mm. Draw vertical line (projector) through b'_2 . Using true length 70 mm as radius and a as centre, draw an arc to intersect the vertical line

- through b'_2 to get b_2 . Join a and b_2 which represents the true length of the line. The inclination ab_2 to XY is the inclination of the line with $VP(\phi)$.
3. Draw the locus of the other end B of the line in top and front views. Draw the locus of b' which is a line passing through b'_1 and parallel to XY line. Also draw the locus of b which is a line passing through b_2 and parallel to XY line.
 4. Rotate the top view ab_1 and front view $a'b'_2$ to the required position. Take a as centre, top view ab_1 as radius, draw an arc to intersect with the locus of b at b . Join a and b to get the top view ab of the line. Take a' as centre, front view $a'b'_2$ as radius, draw an arc to intersect the locus of b' at b' . Join a' and b' to get the front view $a'b'$ of the line.
 5. Check the result obtained by drawing the projector joining b' and b which should be a vertical line.

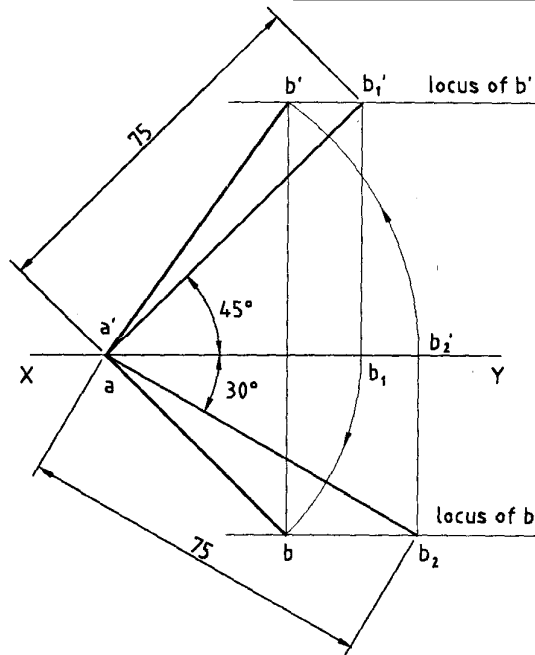
Example

A line AB 75 mm long has its end A in both HP and VP . The line is kept inclined at 45° to HP and 30° to VP .

Solution

Mark the projections of end A by considering it as a point. Its front view a' and top view " a " are marked as a common point on the XY line.

1. Assume that the line is kept inclined to HP and parallel to VP . Draw the front view $a'b'_1$ which is inclined at 45° to XY and has a length of 75 mm. The top view length ab_1 is projected and obtained on XY line.
2. Assume that the line is kept inclined to VP and parallel to HP . Draw the top view ab_2 which is inclined at 30° to XY and has a length of 75 mm. The front view length $a'b'_2$ is projected and obtained on XY line.
3. Draw the locus of b' , passing through b'_1 and parallel to XY line. Also draw the locus of b passing through b_2 and parallel to XY line.
4. Rotate the top view ab_1 by taking " a " as centre, ab_1 as radius to get the intersection point b with the locus of b . Join a and b to complete the top view ab of the line. Rotate the front view $a'b'_2$ by taking a' as centre, $a'b'_2$ as radius to get the intersection point b' with the locus of b' . Join a' and b' to get the front view $a'b'$ of the line.
5. Check the result obtained by drawing the projector joining b' and b which should be a vertical line.



EXAMPLE

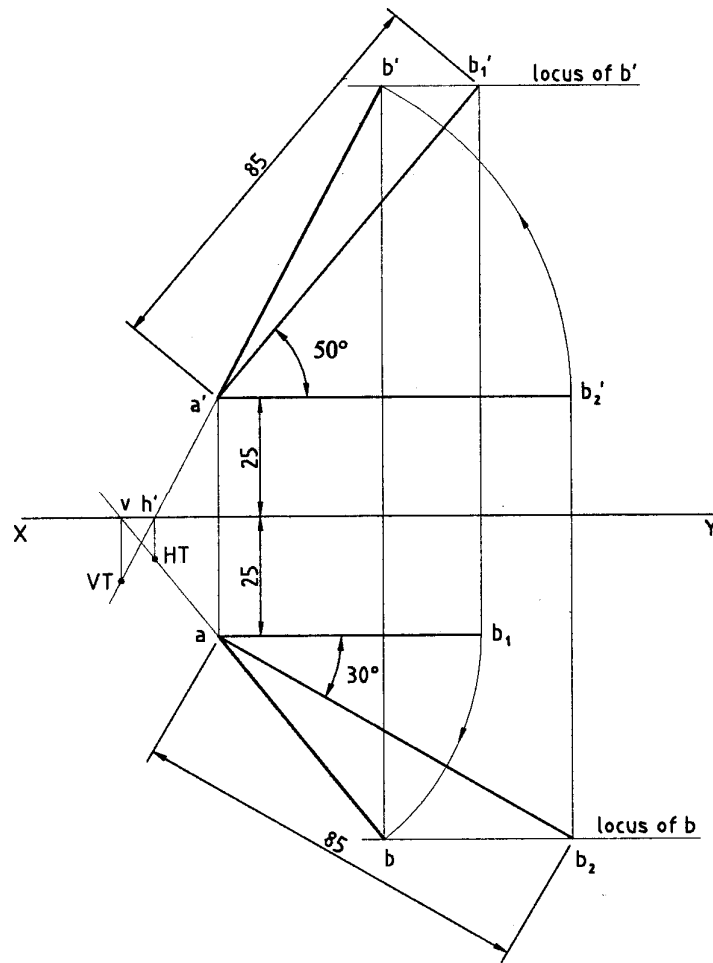
A line AB 85 mm long has its end A 25 mm away from both the reference planes and is in the first quadrant. The line is inclined at 50° to HP and 30° to VP . Draw its projections and mark the traces of the line.

SOLUTION

Mark the projections of end A by considering it as a point. Its front view a' is 25 mm above XY and top view a is 25 mm below XY line.

1. Assume that the line is kept inclined to HP and parallel to VP . Draw the front view $a'b_1'$ which is inclined at 50° to XY line and has a length of 85 mm . The top view length ab_1 is projected and obtained parallel to XY line.
2. Assume that the line is kept inclined VP and parallel to HP . Draw the top view ab_2 which is inclined at 30° to XY line and has a length of 85 mm . The top length ab_1 is projected and obtained parallel to XY line.
3. Draw the locus of b' , passing through b_1' and parallel to XY line. Also draw the locus of b , passing through b_2 and parallel to XY line.
4. Rotate the top view ab_1 to the required position by taking a as a centre, ab_1 as radius to get the intersection point b with the locus of b . Join a and b to complete the top view ab of the line.
5. Rotate the front view $a'b_1'$ by taking a' as centre, $a'b_1'$ as radius to get the intersection point b' with the locus of b' . Join a' and b' to get the front view $a'b'$ of the line.

6. Check the result obtained by drawing the projector joining b' and b which should be a vertical line.



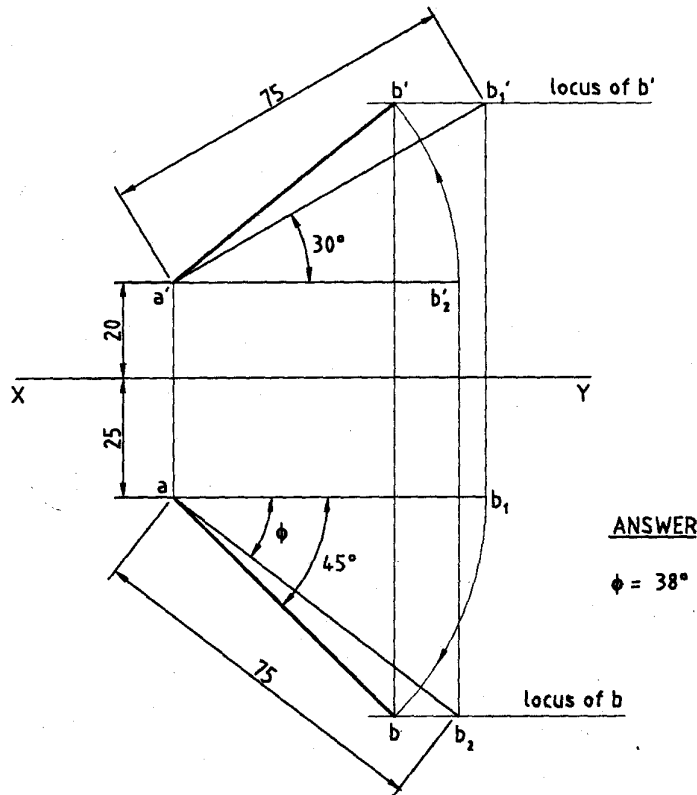
To mark the traces

1. Extended the front view $a'b'$ to get the intersection point h' with XY line.
2. Produce the top view ab to get the intersection point v with XY line.
3. Draw the vertical line from h' to intersect with the top view to get the horizontal trace (HT) of the line.
4. Draw another vertical line from v , to intersect with the front view to get the vertical trace (VT) of the line.

EXAMPLE

One end A of a line AB, 75 mm long is 20 mm above HP and 25 mm in front of VP. The line is inclined at 30° to HP and the top view makes 45° with VP. Draw the projections of the line and find the true inclinations with the vertical plane.

Mark the projections of end of A. Its front view a' is 20 mm above XY and top view a is 25 mm below XY line.



1. Assume that the line kept inclined to HP and parallel to VP. Draw the front view $a'b_1$ inclined at 30° to XY and having a length of 75 mm. The top view length ab_1 is projected and obtained parallel to the XY line.
2. From top view a draw a line inclined at 45° to XY, where the top view b of the line will be obtained. Rotate the top view length ab_1 with the centre a to get the intersection point b . Join a and b to get the top view ab of the line.
3. Draw the locus of b' , passing through b_1 and parallel to XY line. Also draw the locus of b , passing through b and parallel to XY line.
4. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view by drawing an arc with a as centre and radius 75 mm to intersect the locus of b at a point b_2 . Join a and b_2 to get the inclination with VP(ϕ). The front view length $a'b_2$ is projected and obtained parallel to XY line.
5. Rotate the front view $a'b_2$ as radius to get the intersection point b' with the locus of b' . Join a' and b' to get the front view $a'b'$ of the line.
6. Check the result by drawing the projector joining b' and b which should be a vertical line.

Projections of a Line when One End of the Line is in HP and the other in VP

This is considered a special case, when only one condition is given for both ends, one end in HP and another end in VP.

Consider a line AB having its end A in HP, (its position from VP not given) and another end B in VP (its position from HP is not given).

The drawing procedure for this line is the same as in the previous problems, but steps involved in drawing the projections are drawn separately to get the final projections.

The vertical trace (VT) will coincide with the end touching the VP and the horizontal trace (HT) will coincide with the end touching the HP.

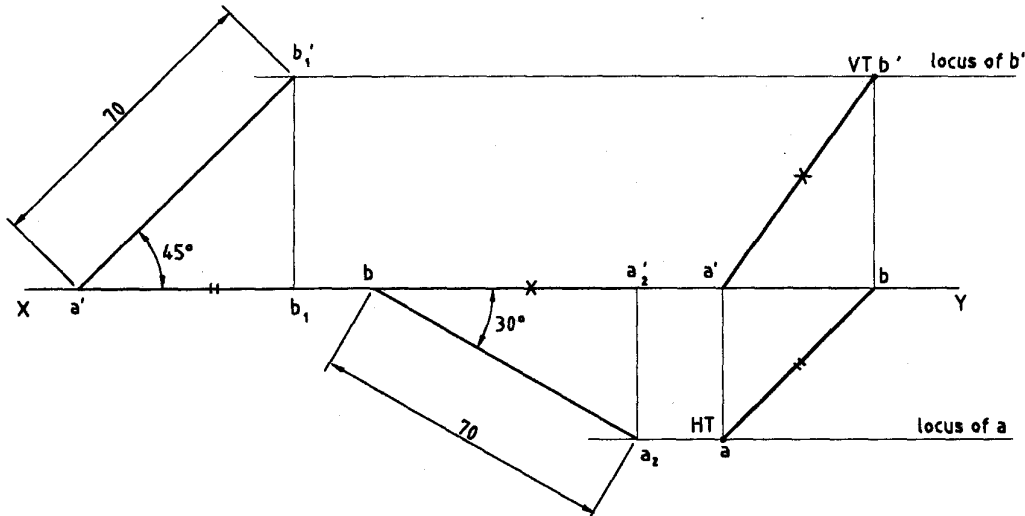
EXAMPLE

A straight line AB is inclined 45° to HP and 30° to VP. The point A is in HP and the point B is in VP. The length of the straight line is 70 mm. Draw the projections of the straight line AB. Also mark the traces.

SOLUTION

1. Mark the front view a' of the end A on XY line arbitrarily. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view $a'b'_1$ of the line which is inclined at 45° to XY and has a length of 70 mm. The top view ab_1 length is projected and obtained on the XY line.
2. Mark the top view b of the end B on XY line arbitrarily. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view ba_2 of the line which is inclined at 30° to XY and has a length of 70 mm. The front view ba'_2 length is projected and obtained on XY line.
3. Draw the locus of b' passing through b'_1 and parallel to XY line. Also draw the locus of a passing through a_2 and parallel to XY line.
4. Mark the front view a' on XY line arbitrarily in another position to get the projections. Considering a' as centre, front view length ba'_2 as radius, draw an arc to get b' in the locus of b' . Join a' and b' to get the front view $a'b'$ of the line. Draw the projector passing through b' to mark the top view b on XY line. Considering b as centre, top view length $a'b'_1$ as radius, draw an arc to get a in the locus of a. Join a and b to get the top view ab of the line.
5. Check the result obtained by drawing the projector joining a' and a which should be a vertical line.

The end A is in HP, so the horizontal trace (HT) is marked coinciding with the top view a of the line. The end B is in VP, so the vertical trace (VT) is marked coinciding with the front view b' of the line.



EXAMPLE

A line AB 85 mm long its end A 25 mm above the HP and 20 mm in front of VP. The end B is 60 mm above HP and 50 mm in front of VP. Draw the projections and find its inclination with HP and VP.

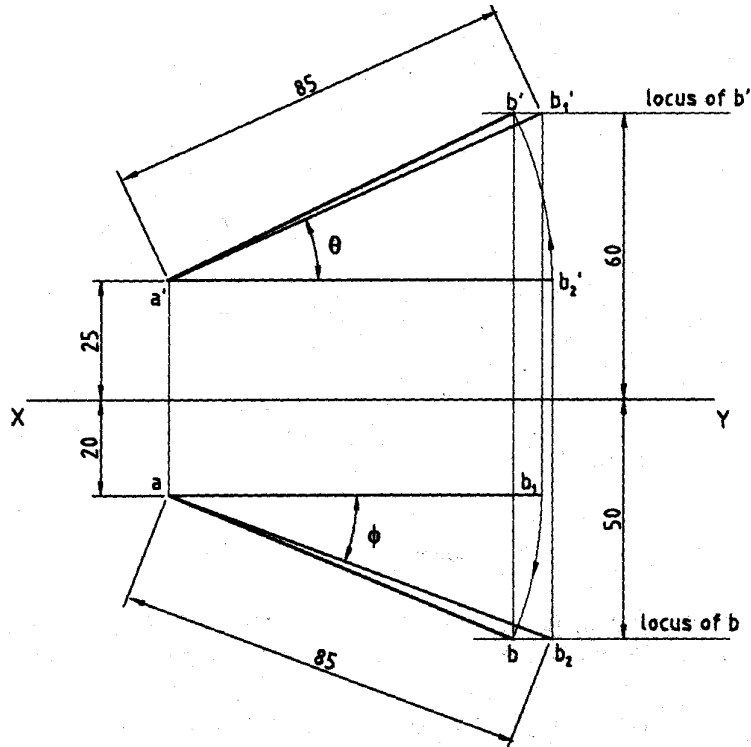
SOLUTION

Mark the projections of end A. Its front view a' is 25 mm above XY and top view a is 20 mm below the XY line.

1. Draw the locus of the other end B in front and top views. Locus of b' is drawn at a distance of 60 mm above the XY line and parallel to it. Locus of b is drawn at a distance of 50 mm below XY line and parallel to it.
2. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view a'b₁' by considering a' as centre and true length 85 mm as radius, cut an arc in the locus of b' to mark b₁' with XY is the inclination of the line with HP(θ).
3. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view ab₂ by considering a as centre and true length 85 mm as radius, cut an arc in the locus of b to mark b₂. The front view length a'b₂' is projected and obtained parallel to XY line. The inclination of top view ab₂ with XY is the inclination of the line with VP(ϕ).
4. Rotate the top view ab₁ to the required position by taking a as centre, ab₁ as radius to get the intersection point b with the locus of b. Join a and b to complete the top view ab of the line. Rotate the front view a'b₂' by taking a' as centre, a'b₂' as radius to get

the intersection point b' with the locus of b' . Join a' and b' to get the front view $a'b'$ of the line.

5. Check the result obtained by drawing the projector joining b' and b which should be a vertical line.



ANSWERS

$\theta = 24^\circ$

$\phi = 21^\circ$

EXAMPLE

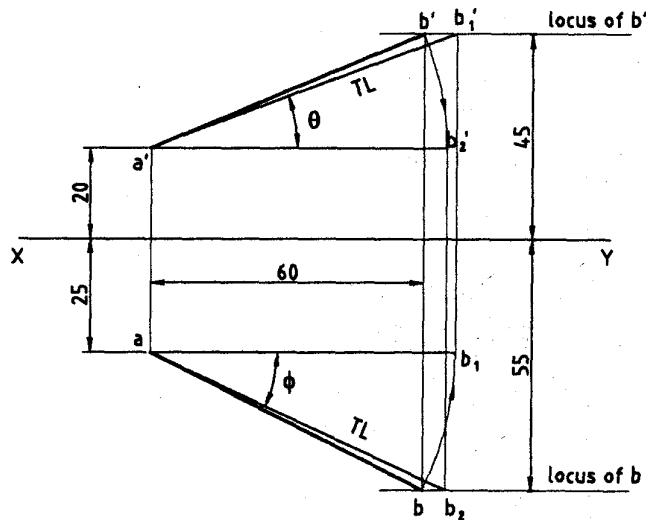
A line AB has its end A 20 mm above HP and 25 mm in front of VP. The other end B is 45 mm above HP and 55 mm in front of VP. The distance between the end projectors is 60 mm. Draw its projections. Also find the true length and true inclinations of the line with HP and VP.

SOLUTION

Mark the projections of the end A. Its front view a' is 20 mm above XY and top view a is 25 mm below the XY line.

1. Draw the projector for the other end B at a distance of 60 mm from $a-a'$. Mark the projection of end B, its front view b' is 45 mm above XY line and top view b is 55 mm below XY line. Join a' and b' to get front view $a'b'$ of the line. Join a and b to get the top view ab of the line.
2. Draw the locus of the end B in front and top views. Locus of b' is drawn passing through b' and parallel to XY line. Locus of b is drawn passing through b and parallel to XY line.

3. Rotate top view ab in the reverse order, take a as centre, top view length ab as radius, draw an arc to get b_1 , on a line drawn parallel to XY line. Project b_1 to locus of b' to get b'_1 . Join $a'b'_1$ which has true length (TL) of the line. The inclination of $a'b'_1$ with XY line is the true inclination of the line with HP(θ).
4. Rotate the front view $a'b'$ in the reverse order, take a' as centre, front view length $a'b'$ as radius, draw an arc to get b'_2 , on a line drawn parallel to XY line. Project b'_2 to the locus of b to get b_2 . Join ab_2 which has true length (TL) of the line. The inclination of ab_2 with XY line is the inclination of the line with VP(ϕ).



ANSWERS

$\theta = 20^\circ$

$\phi = 25^\circ$

TL = 72mm

EXAMPLE

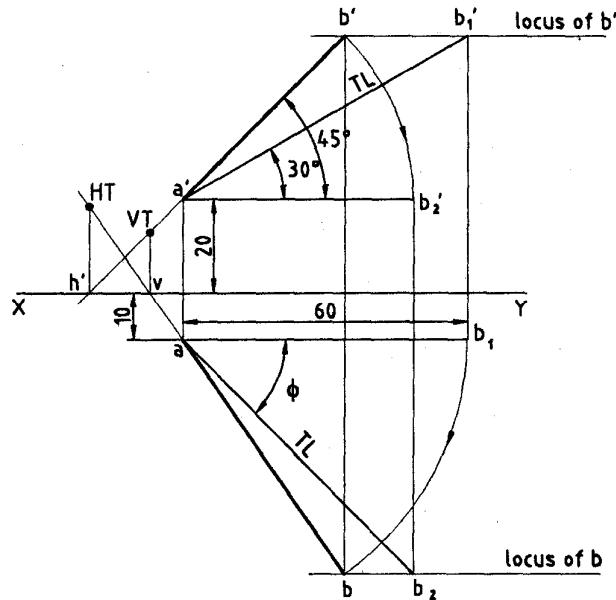
The end A of a line AB is 10 mm in front of VP and 20 mm above HP. The line is inclined at 30° to HP and front view is 45° with XY. Top view is 60 mm long. Draw the projections. Find the true length and inclination with VP. Locate the traces.

SOLUTION

Mark the projections of end A. Its front view a' 20 mm above XY and top view a is 10 mm below XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. Draw its top view ab_1 from a which is parallel to the XY line for a length of 60 mm. Draw a vertical line (projector) from b_1 and draw a line from a' inclined at 30° to XY line, intersecting at b'_1 . The front view length $a'b'_1$ is the true length (TL) of the line.
2. Draw the locus of b' passing through b'_1 and parallel to XY line. Draw the front view $a'b'$ of the line inclined at 45° to the XY line from a' and intersecting the locus of b' at b' .

3. Draw the vertical line (projector) passing through b' . Rotate the top view ab by taking a as centre and ab_1 as radius to intersect with the projector at b . Draw the locus of b passing through b and parallel to XY line.
4. Rotate the front view $a'b'$ in the reverse order. Take a' as centre, front view length $a'b'$ as radius and draw an arc to get b'_2 , parallel to XY line. Project b'_2 to the locus of b to get b_2 . Join ab_2 which has the true length (TL) of the line. The inclination of ab_2 with the XY line is the true inclination of the line with $VP(\phi)$.



ANSWERS

$\phi = 45^\circ$

$TL = 70\text{mm}$

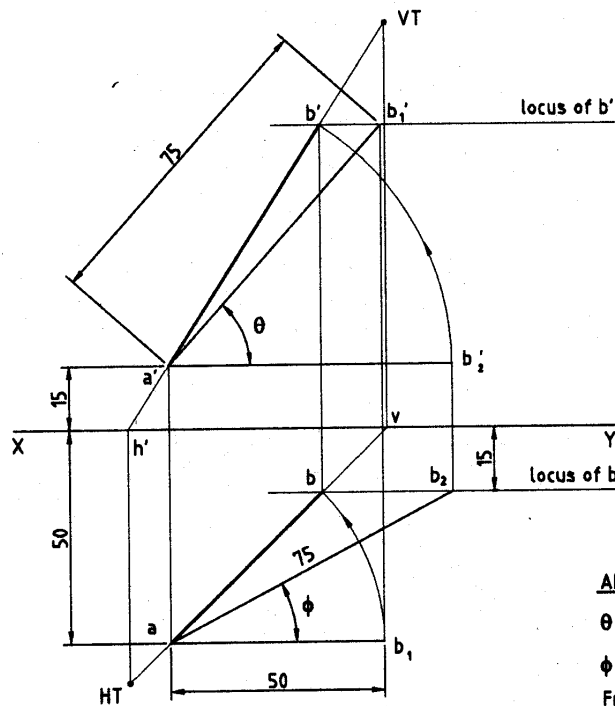
To mark the traces

1. Extend the front view $a'b'$ to get the intersection point h' with XY line.
2. Produce the top view ab to get the intersection point v with XY line.
3. Draw a vertical line from h' to intersect with the top view to get horizontal trace (HT).

Draw another vertical line from v to intersect with the front view to get the vertical trace (VT).

EXAMPLE

A line AB measuring 75 mm long has one of its ends 50 mm in front of VP and 15 mm above HP . The top view of the line is 50 mm long. Draw and measure the front view. The other end is 15 mm in front of VP and is above HP . Determine the true inclinations and traces.



ANSWERS

$\theta = 48^\circ$

$\phi = 28^\circ$

Front view = 66mm

Mark the projections of end A, its front view a' is 15 mm above XY and top view a is 50 mm below XY line. Draw the locus of b , at a distance 15 mm below the XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. The top view ab_1 is marked parallel to XY line for a length of 50 mm. Draw a vertical line (projector) from b_1 . Using true length 75 mm as radius and a' as centre, draw an arc to intersect with the vertical line through b_1 to get b_1' . Join a' and b_1' which represents the true length of the line. The inclination of $a'b_1'$ with XY is the inclination of the line with HP(θ).
2. Assume that the line is kept inclined to VP and parallel to HP. Mark the top view ab_2 , by considering a as centre and true length 75 mm as radius, cut an arc on the locus of b to get b_2 . The inclination of ab_2 with XY is the inclination of the line with VP(ϕ). The front view length $a'b_2'$ is projected and obtained parallel to XY line.
3. Draw the locus of b' which is a line passing through b_1' and parallel to XY line.
4. Rotate the top view ab_1 and front view $a'b_2'$ to the required position. Take a as centre, top view length ab_1 as radius, draw an arc to intersect with the locus of b at b . Join a and b to get the top view ab of the line. Take a' as centre, front view length $a'b_2'$ as radius draw an arc to intersect with the locus of b' at b' . Join a' and b' to get the front view $a'b'$ of the line.

1. Extend the front view $a'b'$ to get the intersection point h' with XY line.
2. Produce the top view ab to get the intersection point v with XY line.
3. Draw the vertical line from h' to intersect the top view to get the horizontal trace (HT) of the line.
4. Draw another vertical line from v to intersect the front view to get vertical trace (VT) of the line.

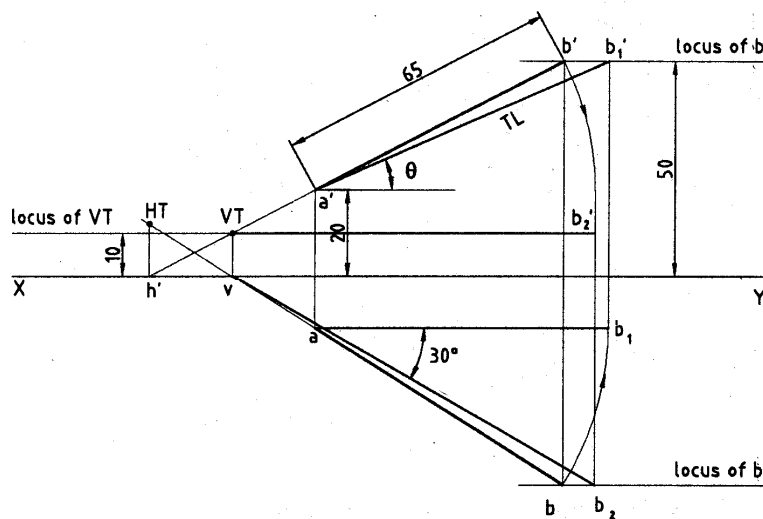
EXAMPLE

A line AB is inclined at 30° to VP has its ends 20 mm and 50 mm above the HP. The length of the front view is 65 mm and its VT is 10 mm above HP. Determine the true length of AB, its inclination with HP and its HT.

SOLUTION

Mark the projection of end A, its front view a' is 20 mm above the XY line.

1. Draw the locus of b' at a distance 50 mm above the XY line. Mark the front view $a'b'$, by considering a' as centre and 65 mm length as radius, cut an arc on the locus of b' to get b' . Join a' with b' to get the front view $a'b'$ of the line. Draw the locus of VT at a distance 10 mm above the XY line. Extend the front view $a'b'$ to mark the VT.
2. Considering the length of the upto the VT where it meets VP. Project the top view of VT which is obtained on XY line and is marked as v . Assume that this line is inclined to VP and parallel to HP. From v , draw its top inclined at 30° to XY line. Rotate its front view VT b' with VT as centre, parallel to XY line to get b'_2 and project it to get b_2 on the inclined line.



ANSWERS

$\theta = 24^\circ$

TL = 75mm

3. Draw the locus of b passing through b_2 and parallel to XY line. Project b' to get b on the locus of b . Join v and b to get the top view vb for the line upto VT . Project a' to get the top view ab of the required line.
4. Assume that the line is kept inclined to HP and parallel to VP . Rotate the top view ab by taking a as centre and ab as radius to get parallel to XY line. Project b_1 to locus of b' to get b'_1 . Join a' and b'_1 to get the true length of the line. The inclination of $a'b'_1$ with XY is the inclination of the line with $HP(\theta)$.

ASSIGNMENT

Q.1. (AMIE W07, 6 marks): Compare first angle projection and third angle projection methods with sketches.

Q.2. (AMIE S05, 10 marks): A magician performs the trick of a floating stick. As seen by a person sitting right in front, as per the orthographic projection rules, the stick has its ends 0.75 m and 1.25m above the floor; appears to be inclined at 30° to floor. The same two ends are found to be 0.25 m and 1.0 m in front of the screen, arranged behind the stick. Using suitable scale, draw the projections of the stick. What is the true length of the stick?

Answer: 1.28 m

Q.3. (AMIE W05, 10 marks): One end of a line of length 60 mm lies at a distance of 20mm in front of V.P and 10 mm above H.P. The line makes 30° with H.P. and the side view measures 45 mm. Draw the projections of the line and determine the inclination with V.P.

Answer: 32°

Q.4. (AMIE W06, 10 marks): A line AB, 60 mm long and inclined at 30° to the ground, has its end A on the ground at 15 mm behind V.P. Its front view measures 45 mm. Draw the top view of the line AB and determine its inclination with the V.P. The H.P. is 45 mm above the ground.

Answer: 42°

Q.5. (AMIE S07, 10 marks): A line AB has its end A 10mm above H.P. and 15mm in front of V.P. The line, 80 mm long, is inclined at 30° to V.P. The distance between the projectors drawn through the end points in top and front views is 55 mm. Draw the projections of the line. Determine inclination with H.P. How far the point B is from the principal planes of projection? Draw its traces and find the distance between the traces measured along xy.

Answer: 33° , 7 mm

Q.6. (AMIE W07, 10 marks): A line AB of 70 mm long has its end A at 10 mm above H.P. and 15 mm in front of V.P. Its front view and top view measure 50 mm and 60 mm, respectively. Draw the projections of the line and determine its true inclinations with H.P. and V.P.

Answer: 30° , 45°

Q.7. (AMIE W07, 14 marks): A line AB, inclined at 30° to the H.P., has its ends A and B, 25 mm and 60 mm in front of the V.P., respectively. The length of the view from above is 65mm and its V.T. is 15 mm above the H.P. Draw the projection of the line AB and locate its H.T.

Q.8. (AMIE W08, 10 marks): The left profile view of a line AB, 80 mm long, makes an angle of 40° to the XY line. Draw the top and front views of the line when length of the left profile view is 50 mm. Take the point A to be 15 mm above H.P. and 60 mm in front of V.P. and the point B being nearer to V.P.

Q.9. (AMIE S09, 10 marks): A room measures 8 m long, 5 m wide and 4 m high. An electric bulb hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the bulb to a switch kept in one of the corners of the room and 1.25 m above the floor. Draw the projections of the wire and determine its true length and slope with the floor.

Answer: 5.2 m, 20°

Q.10. (AMIE S09, 10 marks): A line PQ, 100 mm long, is inclined at 30° to the HP and 45° to the VP. Its mid-point M is in the VP and 20 mm above HP. Draw its projections when its end P is in the first quadrant and Q in the third quadrant.

Q.11. (AMIE W09, 10 marks): The top view of a line is 75 mm long and inclined to XY at 45° . One end is 20 mm above H.P and 10 mm in front of V.P. The other end is 65 mm above H.P. and is in front of V.P. What is the true length of the line and its inclination with H.P. and V.P. ? Also, show its traces.

Answer: 89 mm

Q.12. (AMIE S10, 10 marks): The view from above of a line, AB, inclined at 60° to xy line, measures 85 mm while the length of the view from the front is 65 mm. Its one end A is in the HP and 15 mm in front of the VP. Draw the projections of line AB and determine its true length and true inclinations of it with HP and VP. Find the shortest distance of the mid-point of the line AB from xy line.

Answer: 100 mm, 30° , 50° , 60 mm

Q.13. (AMIE W10, 10 marks): A line PQ has its end, P, 15 mm above H.P. and 10 mm in front of V.P. The end Q is 55 mm above H.P. and the line is inclined at 30° to H.P. The distance between the end projectors of the line when measured parallel to the line of intersection of H.P. and V.P. is 50 mm. Draw the projections of the line and find its inclination with V.P.

Answer: 45°

Q.14. (AMIE S11, 10 marks): Two mangoes on a tree are 5 m and 3 m above the ground and 1.5m and 2.5 m from the central plane of a wall, but on opposite sides of the wall, respectively. The distance between the mangoes measured along the ground and parallel to the wall is 2.5 m. Determine the true distance between the mangoes and the angle of inclination of the line joining the mangoes with the ground and also with the wall.

Answer: 21 m, 24° , 53°

Q.15. (AMIE W11, 10 marks): A line CD, inclined at 25° to the I.P., measures 80 mm in T.V. The end C is in the first quadrant and 24 mm and 14 mm from the H.P. and the V.P., respectively. The end D is at equal distances from both the reference planes. Draw the projections and find the true length and true inclination of the line with V.P. Locate the traces.

Answer: 89 mm, 31°

Q.16. (AMIE S12, 10 marks): A line AB has its ends A and B, 45 mm and 20 mm in front of the VP, respectively. The end projectors of the line AB, when measured parallel to the line of intersection of HP and VP, are 50 mm apart. The HT of the line is 10 mm in front of VP. The line AB is inclined at 35° to the HP. Draw the projections of the line AB and locate the VT. Find the distance of the VT of the line from the HP and the inclination of the line with the VP.

Answer: 20°

Q.17. (AMIE W12, 10 marks): A line AB inclined at 30° to the HP has its ends A and B, 25 mm and 60 mm behind the VP, respectively. The length of the top view is 65 mm and its VT is 15 mm below the HP. Draw the projections of the line and locate its HT. Also, determine the true length of the line AB and true inclination of the line with VP.

Answer: 76 mm, 27°

Q.18. (AMIE S13, 10 marks): The top view of a line AB is 75 mm long and inclined to xy line at 45° . One end A is 20 mm above HP and 10 mm in front of VP. The other end is 65 mm above HP and is in front of VP. What is the true length of line and its inclination with HP and VP? Also, locate its horizontal and vertical traces.

Answer: 90 mm, 31° , 36°

Q.19. (AMIE W13, 10 marks): The top view of a 75 mm long line, CD, measures 50 mm. C is 50 mm in front of the V.P. and 15 mm below (he H.P., 15 mm in front of the V.P. and is above the H.P. Draw the front view of CD and find its inclinations with H.P. and YP. Also, show its traces.

Answer: $\theta = 47^\circ$, 27°